# Application of a Multi-Objective Evolutionary Algorithm to Topological Optimum Design

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**Abstract**

The paper deals with multi-objective Topological Optimum Design (TOD) problems. Considered optimization criteria are the minimization of both the mass of the structure and its maximal displacement under a prescribed loading. Multi-objective evolutionary algorithm using Voronoi representation are applied on the cantilever plate, the popular benchmark problems of TOD. The results are discussed in the light of the single objective approaches used in previous works: whereas the quality of the result is very similar, the number of Finite Element Analyzes needed to obtain a full set of trade-off structures by the multi-objective method is more or less the same that needed to obtain only one solution using a single-objective algorithm with a limit on the maximal displacement: This makes multi-objective approach extremely interesting for solving real life TOD problems where the evaluation cost is usually quite high while the possibility of making of a well justified choice of definitive design is very important.

## Introduction

The general framework of this paper is the problem of finding the optimal shape of a Mechanical structure i.e. a repartition of material in a given *design domain*. In this context, most studies are devoted to minimizing both the mass of structure and its stiffness (or, almost equivalently, on the maximal displacement under a prescribed loading). It is well-known that those objectives are contradictory (i.e. decreasing the weight generally goes through decreasing the stiffness). However, most previous work addressing this problem used single objective optimization method, and either aggregated the objectives, trying to minimize some linear combination fo mass and stiffness, or transformed one of the objectives into a constraint (e.g. minimize the mass with a limit on the maximum displacement).

Nevertheless, the recent raise of evolutionary multi-objective optimization methods allows one to consider both goals simultaneously, and to try to identify the set of optimal trade-offs, also called the *Pareto optimal*) solutions. The foreseen advantages are twofold: First, whereas each single Pareto-optimal solution is the solution of a constrained single-objective problem, not all Pareto-optimal solutions can be obtained using the aggregation method favored by classical deterministic approaches – and a good sampling of all possible trade-offs is mandatory for good decision making. Second, the computational cost of one single multi-objective evolutionary optimization run is of the same order of magnitude than that of one run of a single-objective evolutionary run using the constrained approach: The actual cost of one single trade-off on the Pareto front should thus be much cheaper using the multi-objective approach.

In the present work, a multi-objective evolutionary algorithm is used to solve the two-objective Topological Optimum Design (TOD) problem introduced above. Very few previous studies have tackled that problem [9], but were using the so-called bitarray representation whose drawbacks are well-known. A compact unstructured representation based on Voronoi diagrams [11, 10] is used here, allowing one to search more efficiently the space of discretized structures.

In section 1, some previous studies of the TOD problem are recalled, and a simple TOD benchmark problem is introduced as a two-objective optimization problem. Section 2 presents general issues about multi-objective evolutionary algorithms based on the notion of *Pareto dominance* and, in particular, the NSGA-II approach that is used in this work to solve the two-objective cantilever design problem. Section 3 recalls how the notion of Voronoi diagram can be used to represent a structure in an evolutionary perspective. In section 4, the results of multi-objective optimizations are presented, and compared to those of the single-objective algorithm taken from [11]. Finally, the significance of those results is discussed in the conclusion, and directions for mandatory further work are given.

# 1 Topological Optimum Design

## 1.1 Previous Works

The most up-to-date deterministic approach to TOD is that of homogenization, introduced in [5]. It deals with a continuous density of material in $[0, 1]$. This relaxed problem is known to have a unique solution in the case of linear elasticity and for one single case [4] – and the corresponding numerical method does converge to that non-physical solution [3], which is further forced to a feasible solution (with boolean density). This approach is insofar limited to the linear-elasticity case, and cannot address loadings that apply on the (unknown) actual boundary of the shape (e.g. uniform pressure).

But another limitation is that this method can only handle on single (regular) objective – and the benchmark problems try for instance to minimize a weighted sum of the weight and the stiffness (or *compliance*).

Some limitations of deterministic methods have been successfully overcome by early works using evolutionary computation: in [15, 14] for instance, results of TOD in nonlinear elasticity, as well as the design of an underwater dome (where the loading is applied on the unknown boundary) are presented: all are out of reach for the deterministic methods. But all also deal with a single objective, e.g. minimizing the weight with a constraint on the stiffness.

### 1.2 Two-Objective Cantilever Plate Design

The mechanical model used throughout this paper is the standard two-dimensional plane stress linear model, and only linear elastic materials will be considered (see e.g. [6]). All mechanical figures are dimensionless (e.g. the Young modulus is set to 1) and the effects of gravity are neglected.

The most popular benchmark problem of Optimum Design is the optimization of a cantilever plate: the design domain is rectangular, the plate is fixed on the left vertical part of its boundary (displacement is forced to 0), and the loading is made of a single force applied on the middle of its right vertical boundary. Figure 1 shows the design domain for the $2 \times 1$ cantilever plate problem.

The two objectives are to minimize simultaneously the weight of the structure and the maximal displacement when the given force is applied.

A few recent works address the above two objectives-problem [7, 9]. However, these works, as well as the other works cited here-above, use a binary representation for the structures: section 3.1 will briefly recall its disadvantages.

## 2 Multi-Objective Evolutionary Algorithms

The attention paid to Multi-Objective Evolutionary Algorithms (MOEAs) remarkably increased during the last decade, due to their ability to find multiple trade-off solutions in one single run. The best performing of these approaches are based on the notion of *Pareto dominance*. In addition, they use specific techniques to preserve the diversity, thus giving access to rich and uniform sampling of the Pareto front of the multi-objective problem at hand.

### 2.1 Multi-Objective Optimization

Even when dealing with contradictory criteria, a decision about *only one* final structure must be taken when solving an optimum design problem. It is often the case, however, that some higher-level problem information is available, giving additional decision arguments. There are basically two different ways of handling such arguments: the *preference-based approach* and the *Pareto-based approach*.

The preference-based method needs a quantitative expression of the relative preferences. This allows one to aggregate all criteria into a single function which is then optimized. However, the original problem information is usually qualitative and experience-driven and it may be quite difficult to represent it mathematically.

Moreover, in some particular cases, such representation may lead to completely unexpected results in terms of original preferences. As an illustration, consider a two objective problem *min* $f_1$ and *min* $f_2$ with concave Pareto front (see figure 1 for a plot in the $f_1(x)$, $f_2(x)$ space) Suppose further that we have equal preferences for both criteria. It thus seems quite natural to aggregate $f_1$ and $f_2$ into a weighted sum with equal coefficients. The minimization of the resulting function $f_1 + f_2$ corresponds to moving the lines $f_1 + f_2 = const$ from the origin (it is assumed that the $f_i$s take positive values) toward the positive quarter space until they hit some point of the search space image. Unfortunately, in cases like the one presented in figure 1, such method will always end to either point $A$ or point $B$. Furthermore, it is easy to see that in this case,

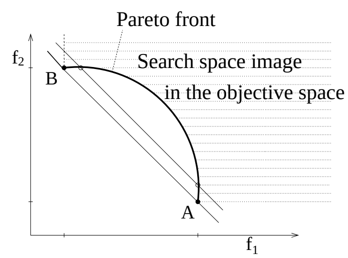



Figure 1: Aggregation approach

*any* combination of preference weights will always return either point $A$ or point $B$. More generally, as the actual shape of the Pareto front is unknown, there is no way to know whether the Pareto front has a concave region, even by looking at the results a posteriori – except by sampling the Pareto front with sufficient accuracy.

The Pareto-based approaches aim at sampling the Pareto set. The problem-specific information is then used to choose among that sample. In another words, these methods allow one to make a choice from a set of best compromise – and that choice can of course be based on original qualitative considerations arising from the problem. This makes Pareto-based approaches less subject to "forgetting" whole regions of possible solutions, like the ones illustrated by figure 1. Pareto-based approaches are hence much more efficient than direct preference-based approaches, especially for design problems for which it is particularly important to make a well justified final decision before moving toward the implementation phase. But of course, practical Pareto-based approaches should have a tractable computing cost, even though their outputs is made of many solutions. This is precisely one of the main advantages of Evolutionary MOAs, that explains why they have become so popular. But before surveying different EMOAs, let us define precisely the key notion of Pareto dominance.

## 2.2 Pareto dominance

Let $x$ and $y$ be two points of the search space; $x$ is said to be *Pareto-dominated* by $y$ if $y$ is not worse than $x$ with respect to all criteria and $y$ is strictly better than $x$ with respect to at least one criterion. All points of the search space that are not dominated by any point of this space compose *Pareto set* of the multi-objective problem at hand. (the image of the Pareto set in the objective space is called *Pareto front* or *Pareto surface*). In another words, *Pareto optimality* is, by definition, a property of all best compromises with respect to the contradictory objectives.

## 2.3 Evolutionary Pareto-Based Algorithms

Multi-objective evolutionary algorithms usually only differ from single-objective algorithms with respect to the Darwinian-like steps of selection and replacement, as the usual relationship *one individual has better fitness than another* does not make sense in a multi-objective context.

Taking into account that the goal is to find a good sampling if the Pareto set of the problem, procedures of selection and replacement are based on the notion of Pareto dominance defined in previous section. However, the Pareto dominance induces only a partial order on the search space, and different ways to turn it into a total order have been designed. For example, for each individual of current population, the number of individuals dominating it gives a scalar value measuring the multi-objective importance of that individual (this value is clearly to be minimized). Another way to measure individuals performance will be given in forthcoming section 2.4.

However, a specific selection mechanism is not enough to obtain a good sampling of the Pareto set: similarly to the single-objective case, finite sampling of all stochastic processes involved in the algorithm creates some genetic drift, tha in turn will result in a converged population. Preserving diversity of solutions is hence particularly important in evolutionary multi-objective optimization. A number of special mechanisms have been designed for this purpose. Most of them require a user-defined parameter like squeeze factor or sharing radius [7]. The NSGA-II approach, detailed below, has been chosen here mainly because it is free of such a parameter.

## 2.4 NSGA-II

Selection and replacement in NSGA-II [8] are based on two hierarchically ordered criteria: in order to compare two individuals (e.g. in a tournament selection process), their domination ranks are first compared – the smaller the better. If their domination ranks are equal, a measure of local sparseness of the search space around each individual, the *crowding distance*, is computed, the larger the better. Standard selection and replacement procedures can then be used, based on that total order. The original NSGA-II uses tournament selection and deterministic replacement among both parents and offspring.

### 2.4.1 Domination rank

The domination rank is based on the Pareto dominance map of the population: individuals that are not dominated by any other individuals in the current population are given rank 1. They are removed from the population and the non-dominated among remaining individuals are given rank 2. This process continues until all individuals are ranked.

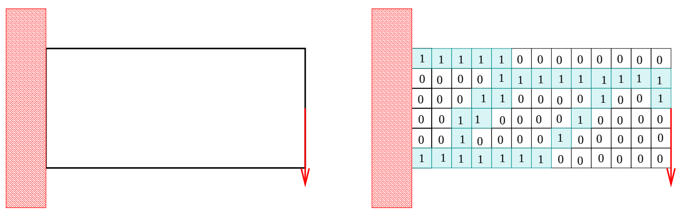
### 2.4.2 Crowding distance

The crowding distance is a measure of the density of solutions in the objective space. Its computation only involves individuals of the same rank, also called partial Pareto front. Each partial Pareto front is sorted according to one objective only. The partial crowding distance for this objective is, for a given individual, the distance between the two neighboring points in this sorted list. The total crowding distance is the sum of these partial crowding distances over all objectives. It can also be viewed as the largest centroid enclosing the individual at hand without including any other point of the same rank.

This measure is not based on any user-defined parameter. However, since it requires sorting the population according to each of the objectives, it becomes computationally heavy for problems with many objectives – and/or large populations.

# 3 Multi-objective Evolutionary TOD

This section discusses the application of an Evolutionary Multi-Objective Algorithm to the TOD problem presented in section 1.2. As always for EAs in general (and hence for EMOAs in particular), the first step is to choose a representation.




(a) The design domain (b) The bitarray representation

Figure 2: The $2 \times 1$ cantilever plate test problem, and a bitarray representation of a structure derived from a regular $13 \times 6$ mesh.

## 3.1 Representations of Structures

One of the most critical decisions made in applying Evolutionary techniques to a particular class of problems is the choice of the representation of the solutions, which determines the search space of the algorithm.

Most previous works that address TOD problems with EAs use the same 'natural' binary representation, termed bitarray[15]: it relies on a discretization into small elements (a *mesh*) of the design domain - the same mesh that is used to compute the mechanical behavior of the structure in order to evaluate its fitness. Each element of the mesh is labeled 1 if it contains material, 0 otherwise (see Figure 1-b).

Though very successful to overcome the main limitations of deterministic methods for TOD problem [15, 16], this representation suffers from strong limitations due to the dependency of its complexity on the underlying mesh of the design domain [11].

These considerations appeal for some more compact unstructured representations whose complexity does not depend on a fixed discretization. Different such alternative representations, that exhibit a self-adaptive complexity, have been proposed [10], and used to solve TOD problem. The rest of the paper will use one of these, namely the Voronoi representation, that will now be presented.

## 3.2 Voronoi Representation

**Voronoï diagrams**: Consider a finite number of points $V_0, \ldots, V_N$ (the *Voronoï sites*) of a given subset of $\mathbb{R}^n$ (the design domain). To each site $V_i$ is associated

(a) The genotype: a list of labeled Voronoi sites. Black dots are sites with label 0 and white dots are sites with label 1.

(b) The phenotype: the Voronoi cells receive the label of the corresponding site, and build a partition of the design domain.

Figure 3: Voronoi representation on a $2 \times 1$ design domain.

the set $Cell(V_i)$ of all points of the design domain for which the closest Voronoï site is $V_i$, termed *Voronoï cell*:

$$Cell(V_i) = \{M \in D / d(M, V_i) = min_{j=1 \cdots N} d(M, V_j)\}$$

where $d(.,.)$ denotes the Euclidean distance function.

The *Voronoï diagram* is the partition of the design domain defined by the Voronoï cells. Each cell is a polyhedral subset of the design domain, and any partition of a domain of $\mathbb{R}^n$ into polyhedral subsets is the Voronoï diagram of at least one set of Voronoï sites (the concept of Voronoi diagrams is a full branch of Algorithmic Geometry[17]).

**The genotype**: Consider now a (variable length) list of Voronoi sites, each site being labeled 0 or 1. The corresponding Voronoi diagram represents a partition of the design domain into two subsets, if each Voronoi cell is labeled as its associated site (see Figure 2).

**Decoding**: However, as some FE analysis is required during the computation of the fitness function, and as re-meshing is a source of numerical noise that could ultimately take over the actual difference in mechanical behavior between two very similar structures, it is mandatory to use the very same mesh for all structures at the same generation. A partition described by Voronoi sites is easily mapped on any mesh: the subset (void or material) an element belongs to is determined from the label of the Voronoi cell in which the gravity center of that element lies.

**Initialization**: a straightforward initialization procedure for the Voronoi representation is a uniform choice of the number of Voronoi sites up to a user-supplied maximum number, a uniform choice of the Voronoi sites in the structure, and a uniform choice of the boolean void/material label.

**Variation operators**: The variation operators for the Voronoi representation are problem-driven:

- The **crossover operator** exchanges Voronoi sites on a geometrical basis. Figure 4 gives an example of application of this operator.
- The **mutation operator** is chosen among the following operators (see Figure 5):

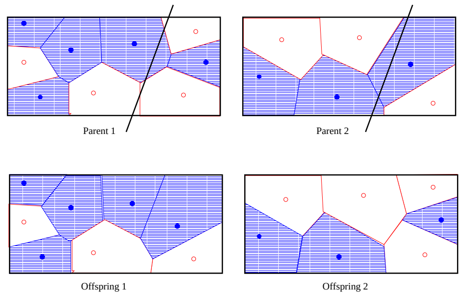



Figure 4: The crossover operator : the same random line is drawn across both diagrams, and the sites on either side are exchanged.

- the *displacement mutation* performs a Gaussian mutation on the coordinates of the sites. As in Evolution Strategies [19], adaptive mutation is used: one standard deviations is attached to each coordinate of each Voronoi site, undergoes log-normal mutation before being used for the Gaussian mutation of the corresponding coordinate.
- the *label mutation* randomly flips the boolean attribute of one site.
- the *add* and *delete mutations* are specific variable-length operators that respectively randomly add or remove one Voronoi site on the list.

In most experiments, once an individual has been chosen for mutation, it has a 50% probability to undergo displacement mutation, and a 16.66% probability to be modified by one of the other three mutations.

### 3.3 Evaluation

The problem tackled in this paper has two objective functions, the weight and the maximal displacement. The computation of the maximal displacement is made using a Finite Element Analysis (FEA) solver [13].

From mechanical considerations, all structures that do not connect the loading point and the fixed boundary are given an arbitrary high "displacement" value without undergoing any FEA. Moreover, the material in the design domain that is not connected to the loading point – and thus has no effect on the mechanical behavior of the structure – is discarded during the FEA and only slightly penalizes the weight. (see [15] for more numerical details).

## 4 Experimental Results

This section presents experimental results on cantilever benchmark problems, and discusses the comparatives results obtained for two-objective and single-objective constraint problem.

### 4.1 Evolutionary Experimental Conditions

The experiments have been performed using the following settings: the maximum number of Voronoi sites allowed per structure is set to 40; the population

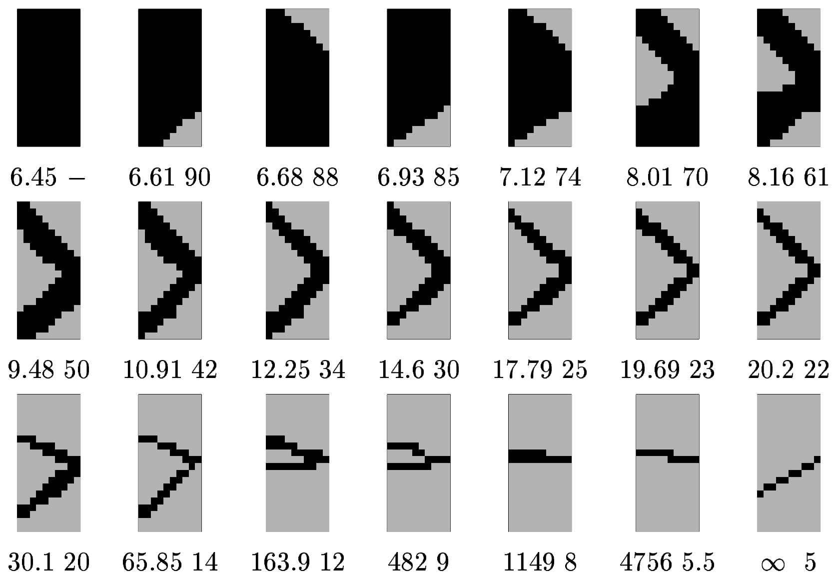



Figure 5: 21 trade-off shapes for the $1 \times 2$ cantilever plate discretized using $10 \times 20$ regular mesh. Under each shape the maximal displacement and the weight (in %) are given. The last structure is mechanically almost infeasible, and has a very large displacement.

size is fixed to 300 and the maximum number of generations to 400. The following variation operators were used: intermediate crossover is applied with probability 0.7, and the mutation rate (per individual) is 0.2; Relative weights among the different mutations are 1/2 for the displacement mutation and 1/6 for the three other mutations. Replacement and selection are those of the original NSGA-II method described in section 2.4, with tournament size 2. All CPU times are given related to a Pentium III processor running at 800MHz under Linux.

## 4.2 Discussion

The proposed approach was applied to the $1 \times 2$ and $2 \times 1$ cantilever plate benchmark problems, repectively discretized into a $10 \times 20$ and $20 \times 10$ regular meshes, to stay within reasonnable computing times.

Figures 5 and 7 display a set of shapes (solutions) selected from the results a respectively 3 and 2 runs, and having a wide range of trade-off between the weight and maximal displacement. The structures presented in these figures are ordered according to their weights from left to right and from top to bottom, starting with the full structure. Note that the extremely light structures do not make sense from the mechanical point of view as the underlying discretization is not fine enough in order to get significant results.

Figures 6 and 8 show the corresponding Pareto fronts for both problems. Again, a few runs were necessary to obtain a good sample of those Pareto fronts, as each one of them was better at sampling a given area of the front. It

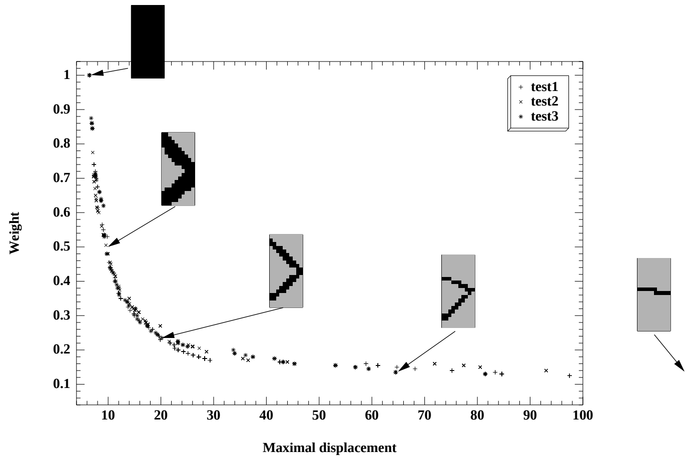



Figure 6: Results for the $1 \times 2$ cantilever: The Pareto fronts obtained after 400 generations with 300 individuals from three independent runs (labeled test1, test2 and test3).

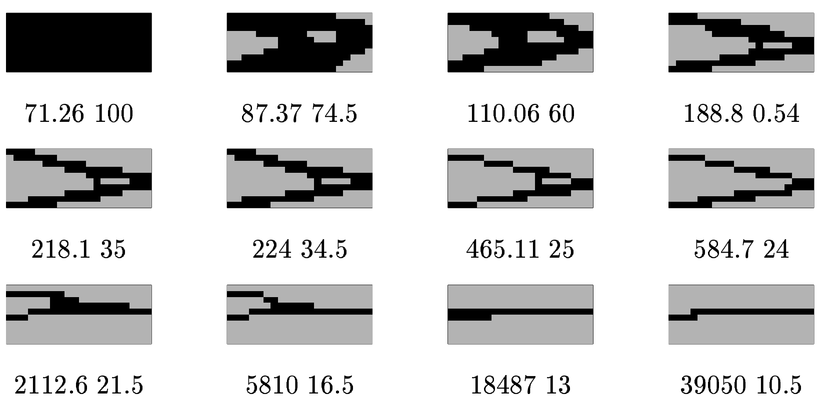



Figure 7: 12 trad-off shapes for the $2 \times 1$ cantilever plate discretized using $20 \times 10$ regular mesh. Under each shape the maximal displacement and the weight (in %) are given.

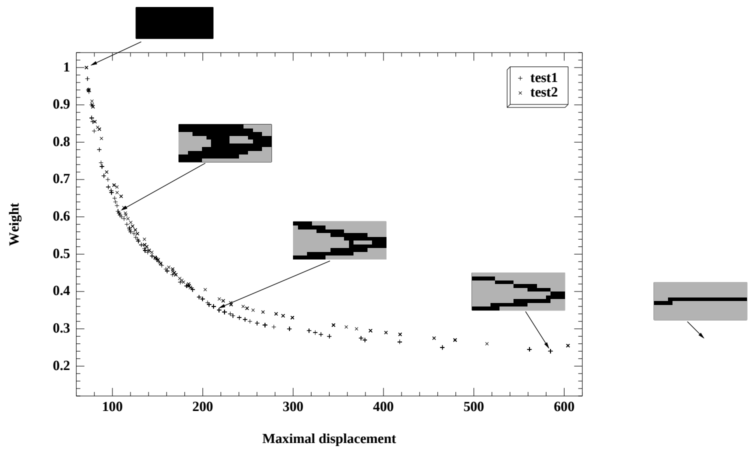



Figure 8: Results for the $2 \times 1$ cantilever: The Pareto front obtained after 400 generations with 300 individuals of two independent runs (labeled test1 and test2).

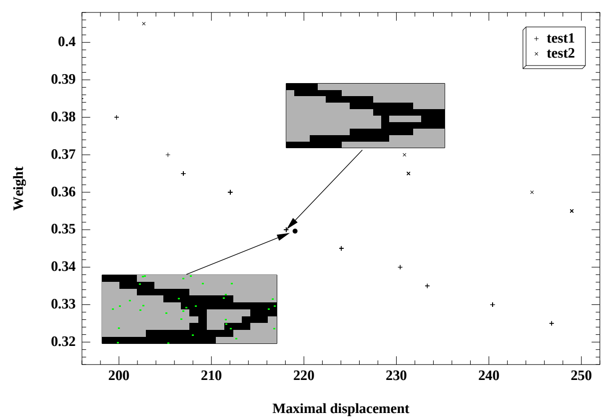



Figure 9: Zoom on the Pareto front in the neighborhood of maximal displacement equal to 220. The top structure has been obtained by the multi-objective algoritm, while the bottom structure is the best result from the single-objective contrained approach [10].

seems that some progress is still necessary, for instance in the niching method, in order to get a good sample from one single run - or maybe a larger population is needed.

Looking more closely at the Pareto front of figure 8, and zooming in the neighborhood of maximal displacement 220 (figure 9) one can notice that the MOEA found a structure very similar to that obtained for single-objective constrained problem in [10]. Comparing now the CPU times, one run of the single-objective constrained approach took 27 minutes, while 55 minutes were needed by one multi-objective run. But on the other hand, multi-objective approach needed approximately 100000 fitness computations to obtain the set of trade-off structures of the same quality that the single objective constrained solution found after around 130000 evaluations. Hence the difference in CPU time is due to the multi-objective specific treatments, like Pareto-ranking and crowding-distance computation. It can thus be expected that this difference will vanish, and might even turn the other way round, if the cost of the FEAs increases, which will be the case for real-world problems with very fine meshes.

# 5 Conclusion and Future Work

The optimization results for a simple benchmark presented in this work, should be considered as a proof of principle of the application of a multi-objecive evolutionary algorithm combined with Voronoi representation to a TOD problem. In that respect, these results are a success: for each test case, the few dozen structures that are obtained as a sample of the Pareto front are very similar to the ones obtained using a single-objective constrained approach – on run for each structure. If the goal is to obtain such a sample, then we can claim at least one order of magnitude improvement in the overall computing effort. However, as already mentionned, some effort will be made to tune the niching strategy in order to better sample the Pareto front.

Moreover, Pareto-based evolutionary algorithms do open up new perspectives in TOD. First, MOEAs will be coupled with other high level structure representations, like the other representations defined in [10], or more advanced representations based on embryogenies that would widen the search to hierarchical and modular structures [12]: it is hoped that even more accurate results will thus be gathered.

Second, there are many other problems in TOD than the simple weight vs stiffness optimization: modal optimization will be the first domain where MOEAs will certainly bring some improvement, as modal shape optimization is by essence multi-objective (optimizing only the eigenfrequencies does not make much sense). Moreover, evolutionary optimization will for instance also allow to avoid certain ranges of eigenfrequencies (e.g. in car industry, to avoid sea-sickness), and not only to maximize the first one [1]. Another important kind of problem where multi-objective optimization can be very useful is that of multi-loading optimization: at the moment, the only results dealing with multi-loading optimization use an agreagation method, be they deterministic [2] or evolutionary [15], whereas dealing with each loading case as separate objective will allow much more flexibility in the final design.

Nevertheless, some studies will also necessary to try to focus the multi-objective search. Indeed, while the theoretical Grail of multi-objective optimization is to obtain the whole Pareto front of the problem at hand, it has two practical drawbacks. First, some computational effort is wasted evaluating totally useless trivial solutions (e.g. the extreme solutions in figure 5, with either 100% or almost 0% weight). But more important, when many objectives are involved, trying to sample the whole Pareto front does not result in any interpretable result due to the complexity of analysis in large high dimensional spaces [18]. On-going work tries to address this issue that is critical for real-world applications.

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